

A SARIMA and Adjusted SARIMA Models in a Seasonal Nonstationary Time Series; Evidence of Enugu Monthly Rainfall

D MADHUSUDANA REDDY, R SIVA GOPAL, B VENKATA RAMANA
PROFESSOR^{1,2}, ASSISTANT PROFESSOR³

madhuskd@gmail.com, sivagopal222@gmail.com, bvramana.bv@gmail.com

Department of Mathematics,

Sri Venkateswara Institute of Technology,

N.H 44, Hampapuram, Rappthadu, Anantapuramu, Andhra Pradesh 515722

Abstract

In a normal stationary series with a seasonally nonstationary underlying variable, the study contrasts SARIMA with adjusted SARIMA (ASARIMA). Based on actual rainfall data and the Box-Jenkins iterative process that determines the best fit, AIC selected ASARIMA(2,1,1)₁₂ from among the eleven SARIMA sub-classes and seven ASARIMA model sub-classes. This was in preference to all of the discovered SARIMA(p,0,q)x(P,1,Q)₁₂ sub-classes. Up to the 48th lag, the diagnostic test shows no autocorrelation. The fitted model produces very similar projected values to the actual results. Since ASARIMA penalises parameter redundancy and excessive sum of square errors, it is suited for regular stationary time series with seasonal features.

Keywords: Wet weather, AIC, ASARIMA model, seasonal nonstationary time series, and rainfall.

I. Introduction

According to [7], monthly or quarterly data that exhibit predictable seasonal movements should be processed using seasonal autoregressive integrated moving average

(SARIMA) terms. The source given above might be consulted for technical specifics. An example of a situation where seasonal differencing is necessary is when the underlying variable of interest exhibits a cyclical pattern that is not stationary according to the regularity of the data. Instead of using the SARIMA(p,d,q)x(P,D,Q) model, the Adjusted SARIMA(P,D,Q) model is more appropriate for time series variables that exhibit these traits. But, at higher lag orders, the autocorrelation of the model's residuals could be considerable, and SARIMA(p,d,q)x(P,D,Q)s raises the sum of square residuals for such time series because of certain redundant parameters. The benefits of the Adjusted SARIMA model over the SARIMA model are as follows. When it comes to representing parameters, adjusted SARIMA models are thrifty. One of the most influential natural variables influencing agricultural output in Nigeria and throughout the world is rainfall. Agriculture and the state's economy are very sensitive to rainfall variability and the pattern of extremely high or low precipitation. Because of the increasing likelihood of catastrophic flooding at the height of the rainy season, the pursuit of more study on the topic has been accelerated by climate change on a worldwide scale.

Among the eastern Nigerian states, Enugu State occupies a tropical rain forest zone

bordered by derived savannah at the base of the Udi Plateau. In the south, you'll find Abia and Imo states; in the east, Ebonyi; in the northeast, Benue; in the northwest, Kogi; and in the west, Anambra. Located at an elevation of 223 metres (732 feet) above sea level, Enugu has favourable weather conditions and soil-land characteristics throughout the year. Even when it rains, the soil drains efficiently. March through November has the highest humidity in Enugu's tropical rain forest zone, which is characterised by a derived savannah [13]. Average daily temperatures in Enugu State are 26.7 °C (80.1 °F). Enugu State has its warmest month, February, with an average temperature of about 87.16 °F (30.64 °C), and its coldest month, November, with an average temperature of 60.54 °F (15.86 °C). Typical rainfall ranges from a low of about 0.16 cc (0.0098 cu in) in February to a high of around 35.7 cc (2.18 cu in) in July. The 2006 census revealed a population of 3,267,837 in Enugu State, with a 2012 estimate of more than 3.8 million. Precipitation pattern modelling and forecasting has received a lot of interest from researchers. As an example, monthly rainfall in Tamil Nadu, India, was fitted using a SARIMA(0, 1, 1)x(0, 1, 1)₁₂ model in [14]. For monthly rainfall in Kuantan, Malaysia, and Malaaca, the SARIMA models of orders (4, 0, 2)x(1, 0, 1)₁₂ and (1, 1, 2)x(1, 1, 1)₁₂, respectively, were fitted in [16]. Using data from 1975 to 2009, the SARIMA model was evaluated for its suitability in predicting rainfall in the Brong Ahafo (BA) Region of Ghana [1]. The data showed that January, December, and February had the least amount of precipitation, while September and October have the most. The Brong Ahafo Region of Ghana's monthly average rainfall numbers were predicted using the SARIMA (0,0,0)x(1,1,1)₁₂ model. With the use of the seasonal SARIMA (5, 1, 0)x(0, 1, 1)₁₂ model, the monthly rainfall in

Port Harcourt, Nigeria was determined in [12]. No discernible pattern can be seen in the time-plot. The graphic clearly shows the predicted and recognised seasonality. After performing seasonal (i.e. 12-point) differencing on the data, the changes across seasons are re-analyzed using a nonseasonal differencing method. The predicted 12-month seasonality, together with the participation of a seasonal moving average component and a nonseasonal autoregressive component of order 5, are shown by the correlogram of the resulting series. This leads us to the previously mentioned model. It has been shown that the model is adequate. Quarterly rainfall in Port Harcourt, Nigeria was simulated as a SARIMA(0, 0, 0)x(2, 1, 0)₄ equation in [15]. [3] used monthly rainfall data from 2004–2015 to analyse rainfall in Oshogbo, Osun State, Nigeria. A high degree of volatility, defined by seasonal and irregular fluctuations, is seen by the time plot of the rainfall data. Next, the logistic model was used to predict precipitation for the following two years, and it proved to be superior. Researchers in Port Harcourt, Nigeria, used the ARMA(p,q) model to study the average yearly rainfall pattern [5]. The rainfall data used spans from 1981 to 2016. The optimal sub-classes of ARMA(p, q) were chosen using sum-of-squares deviation forecast criteria (SSDFC).

all of the information. According to SSDFC's evaluation, the highest performing model was ARMA(1, 2), outperforming ARMA(2, 1) and ARMA(2, 2) models. Both AIC and BIC gave their stamp of approval to the chosen model. In the end, we found that ARMA(1, 2) is a useful tool for predicting the long-term water quality for agricultural and hydrological purposes, as well as for raising awareness about the need of flood control in Port Harcourt. Utilising univariate monthly rainfall data ranging from 1981M1–2017M12, the seasonal autoregressive integrated moving average

(SARIMA) model was used to simulate the monthly rainfall pattern in Imo state [6]. We compared nine(9) distinct sub-classes of the twelve (12) models found using the sum of square deviation forecast criteria (SSDFC). More specifically, the results showed that SARIMA(0,0,0)x(1,1,1)₁₂ is the better fit for forecasting the state's monthly rainfall. For several reasons, including managing flood

$$A(L)\nabla^d X_t = B(L)u_t$$

risks, irrigation, surface water, and more, it is crucial to predict monthly rainfall in every state in order to understand the geographical and temporal variability. In addition, modelling the seasonal pattern of rainfall in the state is essential for agricultural planning due to the need to diversify Nigeria's economy towards an agricultural foundation. So, to predict the amount of rain that would fall in Enugu State, this research compares two models: the seasonal ARIMA (SARIMA) and the adjusted seasonal ARIMA (ASARIMA). A basic analytical model modified from the SARIMA process is presented in the paper. Section two covers the procedures and materials used in the study, section three details the analysis and outcomes of the data,

$$A(L)\Phi(L^s)\nabla^d\nabla_s^D X_t = B(L)\Theta(L^s)u_t$$

and section four concludes the work.

II. MATERIALS AND METHODS

This section highlights the methods and sources of data collection, variable measurement, method of unit root test, model specification, and model identification, method of data analysis, model comparison techniques and diagnostic checks.

A. Source of Data and Variable Measurement

The monthly rainfall data was obtained from central bank of Nigeria (CBN) (2018)

statistical bulletin. The univariate time series data collected covered the period of 1981M1- 2017M12 (432 observations of monthly rainfall data). Rainfall is usually measured in millimetre using rain gauge.

B. SARIMA Model Specification

If the time series X_t is nonstationarity due to the presence of one or several of five

(1)

conditions: outliers, random walk, drift, trend, or changing variance, it is conventional that first or second differencing (d) is necessary to achieve stationarity. Hence, the original series is said to follow an autoregressive integrated moving average model or orders p, d and q denoted by ARIMA(p, d, q) of the form

If the series X_t exhibits seasonal patterns of nonstationarity, this may be detected using time plot, correlograms or even unit root test. And according to [7] Seasonal ARIMA

models sometimes called SARIMA models

has the general form SARIMA(p, d, q)

(P, D, Q)_s and it is given as

where A(L) is the autoregressive (AR) operator, given by $A(L) = 1 - \alpha_1 L - \dots - \alpha_p L^p$ and B(L) is the moving average (MA) operator, given by $B(L) = 1 - \beta_1 L - \dots - \beta_q L^q$

$\beta_q L^q$. For L denotes the backshift

operator. $\Phi(L^s)$ and $\Theta(L^s)$ are lagged

seasonal AR and MA operators of order P and Q respectively. The operator ∇^d denotes the difference operator defined by $\nabla^d = 1 - L$ and $d \leq 2$. The

∇_s^D represents the seasonal difference operator defined by $\nabla = 1 - L^s$ and D is the seasonal differencing order. The seasonal differencing $(1 - L^s)$ is called the simplifying operator, which renders the residual series stationary and amenable to further analysis.

operators of order P and Q. In such cases, it is appropriate to assume that $A(L)d=0$ so that (2) can be of the form;

$$\Phi(L_s)\nabla_s X_t = \Theta(L_s)$$

where $\Phi(L_s)$ is the seasonal autoregressive (SAR) operator, given by
and $\Theta(L)$ is the seasonal moving average (SMA) operator, given by

$$\begin{aligned} \nabla_s X_t = \omega + \phi_1 \nabla_s X_{t-(s \times 1)} + \dots + \phi_P \nabla_s X_{t-(s \times P)} \\ + \theta_1 u_{t-(s \times 1)} + \dots + \theta_Q u_{t-(s \times Q)} \end{aligned} \quad (4)$$

.Generally, the Adjusted SARIMA(P,D,Q)s model which hereafter is known as ASARIMA(P,D,Q)s model with the inbuilt constant term is specifically of the form;

where ω is the constant parameter and s is the seasonal index. ASARIMA(P,D,Q)s model is special case of SARIMA(p, d, q) $\times (P, D, Q)_s$ model.

D. Model Identification

The ACF of an MA(q) model cuts off after lag q whereas that of an AR(p) model is a combination of sinusoidals dying off slowly.

C. Adjusted SARIMA Model

The SARIMA model in (2) is the combination of nonseasonal AR and MA operators of order p and q and seasonal AR and MA operators of order P and Q. If a univariate time series is stationary in non-seasonal component (where d=0) and exhibits a purely seasonal pattern that is nonstationary (where D=1). It could be parsimoniously better to only fit the seasonal AR and MA

B(L) 1 and

On the other hand, the PACF of an MA(q) model dies off slowly whereas that of an AR(p) model cuts off after lag p. AR and MA models are known to exhibit some duality relationships. Parametric parsimony consideration in model building entails the use of the mixed ARMA fit in preference to

either the pure AR or the pure MA fit.

Note that SARIMA can be fitted irrespective of whether the underlying variable is seasonally stationary or not. The differencing operators $d = 0$ for stationary series and for nonstationary series d could

$$\nabla y_t = \alpha + \alpha_1 t + \beta y_{t-1} + \sum_{i=1}^k \xi_i \nabla y_{t-i} + a_t \quad (5)$$

be 1 or 2 depending on the order of integration of the variable under study. The seasonal difference D may be chosen to be at most equal to 1. The nonseasonal and seasonal AR orders p and P are fitted by the nonseasonal and the seasonal cut-off lags of the partial autocorrelation function (PACF) respectively. Similarly the nonseasonal and the seasonal MA orders q and Q are fitted respectively by the nonseasonal and seasonal cut-off points of the ACF.

E. Conditions for ASARIMA(P,D,Q)s Model

The following conditions should lead to the adoption of ASARIMA(P,D,Q)s model;

- 1) The underlying univariate time series must be non-seasonally stationary ($d=0$) and exhibits cognizable seasonal pattern. Note, seasonal differencing (D) may be 0 or 1.
- 2) The ACF must reveal seasonal oscillation with significant spikes at every k th lag, here $k = s \times i$ and $i = 1, 2, \dots, K$.
- 3) The PACF tends to cut-off at every k th lag and cut-in.
- 4) If the spikes in (iii) tails off at every k th lag consider fitting ASARIMA(P,D,0)s
- 5) If the spikes in (iii) do not tails off at every k th lag consider fitting ASARIMA(0,D,Q)s

- 6) If the spikes indicate mixture of (iv) and (v) consider fitting ASARIMA(P,D,Q)s
- 7) Use some information criteria such SSDFC, AIC BIC, SC etc to select the best fitted model.

F. ADF Unit Root Test

ADF unit root test helps to check the order of integration of the variables under study. The unit root test here, is based on Augmented Dickey Fuller (ADF) test and is of the form

where k is the number of lag variables. In (5) there is intercept term, the drift term and the deterministic trend. The non deterministic trend term removes the trend term in (5). And it can be carried out with the choice of removing both the constant and deterministic trend term in the above regression.

ADF unit root test null hypothesis $H_0 : \beta = 0$ and alternative $H_a : \beta < 0$. According to [7], if the ADF test statistic is greater than 1%, 5% and 10% critical values, the null hypothesis of a unit root test is accepted. ERS unit root test will used to consolidate the result provided by ADF test. See the technical details in [11].

G. Model Comparison

There are several model selection criteria in literature such as; Bayesian information criterion (BIC), Aikaike information criterion (AIC), residual sum of squares and so on. If n is the sample size and RSS is the residual sum of squares, then, BIC and AIC are given as follows;

$$BIC = 2k + \ln(RSS)$$

$$AIC = 2k + n \ln(RSS)$$

In this context, n denotes the sample size, k denotes the number of estimated parameters (or regressors in the case of regression), and RSS stands for the residual sum of squares determined by the estimated model. Be aware, however, that the amount of parameters used to estimate a model has an effect on both BIC and AIC . When it comes to BIC , free

$$SSDFC = \frac{1}{m} \sum_{i=1}^m (y_{t(l,i)} - \hat{y}_{t(l,i)})^2 \quad (8)$$

parameters are penalised, while AIC shrinks with an increasing number of free parameters that need estimation. Model selection, however, will be based on AIC for this investigation. To evaluate the efficacy of the models' outputs across a 150-day prediction horizon, we will use the sum-of-squares deviation forecast criteria proposed in [4]. Plus, it takes the shape

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III. In this context, l represents the lead time, m is the intended number of

deviations between the actual and forecast values (which should be reasonably large), $y_{t(l,i)}$ represents the actual values of the time series that correspond to the i th position in the forecast, and $\hat{y}_{t(l,i)}$ represents the forecast values that correspond to the i th position in the actual values. When comparing models, the one with the lowest $SSDFC$ value is the best performing model in terms of output and can capture the underlying fitted model's behaviour as closely as possible. Estimation of Models (H) An iterative approach is used to estimate the coefficients by calculating the least squares. The back predictions and sum of squares error (SSE) are computed at each iteration point. View [8] for more information.

IV. DATA ANALYSIS AND RESULTS

This section presents the time series plot of Enugu monthly rainfall data, results of ADF unit roottest, plots of ACF and PACF and estimates of SARIMA(p,d,q)×(P,D,Q)s model.

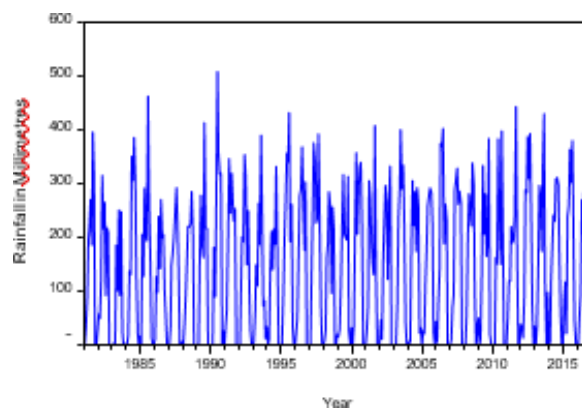


Fig.1. Time plot of EMR (1981M1 – 2016M12)

The plot of monthly rainfall in Figure 1 exhibits seasonal nonstationary pattern. It is also observable that the time series plot lacks trend with the highest precipitation of 508.3 Millimeters in July 1990 and lowest precipitation of 0.5 Millimeters in January and February the same year.

the results of the ADF and ERS unit root tests in Table I above. Accordingly, the investigated monthly rainfall is not changing. The SARIMA model, which takes into account the stationarity of the EMR variable, will be applied to the data.

Section A. Correlogram

Figures 3 and 4 below show the autocorrelation function (ACF) and partial

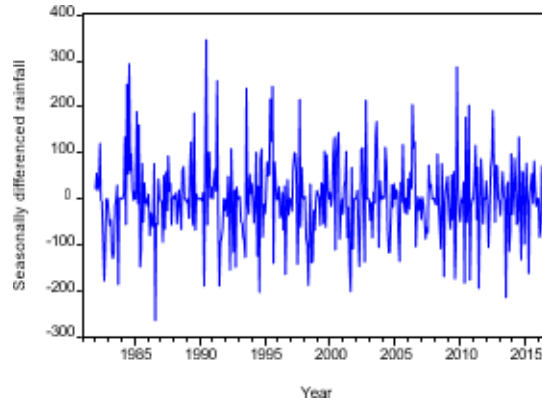


Fig.2. Time plot of Seasonally differenced EMR (1981M1 – 2016M12)

autocorrelation function (PACF) plots, respectively, used in the correlogram for model identification.

The seasonally differenced EMR data in Fig.2 is seasonally stationary with most of the data concentrated around zero.

TABLE I. ANALYSIS OF ORDER OF INTEGRATION OF EMR

Test	Rainfall	DT	Lags	Test Value	Critical Values	Remark
					1% 5% 10%	
ADF	EMR	C	11	-3.8511	-3.4457 -2.8682 -2.5704	I(0) significant under 5%
ERS	EMR	C	5	2.4747	1.9900 3.2600 4.4800	I(0) significant at 5%

Keep in mind that DT stands for "deterministic term."

The EMR variable is integrated order zero I(0), significant at the 5% level, according to

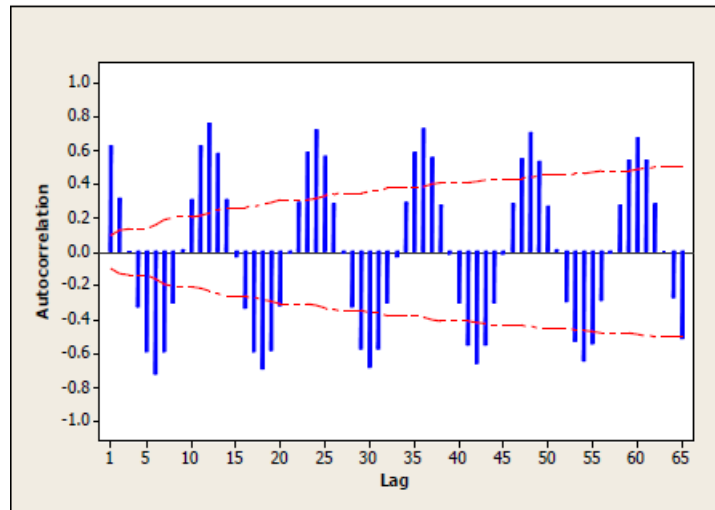


Fig.3. Plots of ACF EMR (with 5% significance limits for the correlogram)

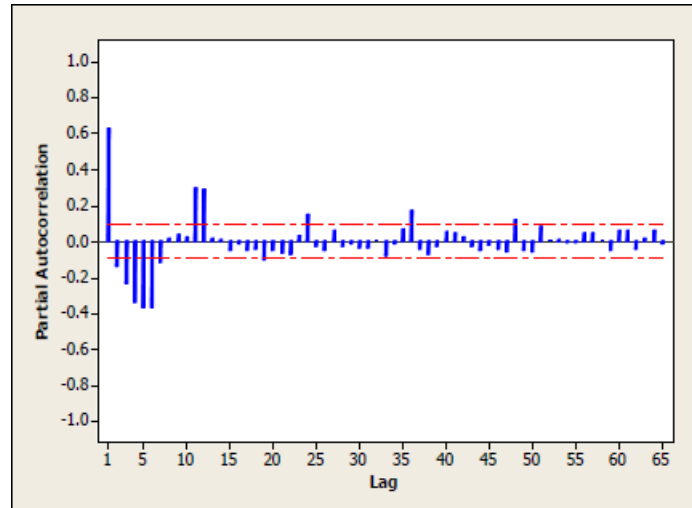


Fig.4. Plots of PACF for EMR (with 5% significance limits for the correlogram)

Figure 3, which shows the autocorrelation function plot, shows that there is a seasonal influence. A seasonal ARMA model might be a good match for the rainfall data, according to the cyclical correlogram with a seasonal frequency. This finding supports the idea that the model should account for seasonal differences. Rainfall exhibited seasonality as seen by the time plot. However, in cases when the time plot fails to adequately illustrate this, the autocorrelation function (ACF)—which

has a considerable lag—could disclose the value of s . Figures 3 and 4 illustrate that the ACF and PACF surge once a year or every twelve months. Seasonal nonstationarity is readily shown by the ACF. At 12, 24, 36, and 48 delays, the seasonal surges are shown by the PACF. Figure 4's ACF shows seasonal nonstationarity as the peaks gradually fade throughout the seasons. The recurring peaks at every 12th lag up to 48th lag, evoking seasonal differencing at lag 12, demonstrate the 12-month PACF periodicity.

A. Model Comparison

This section presents a comparison of 27 possible models using SSDFC as presented in Table II below;

TABLE II. MODEL SELECTION USING AIC

Model	AIC	BIC	SSDFC
SARIMA(1,0,0)×(1,1,1) ₁₂	3557.36	16.2723	3114.06*
SARIMA(2,0,0)×(1,1,1) ₁₂	3559.35	18.2863	3116.75
SARIMA(3,0,0)×(1,1,1) ₁₂	3560.40	20.2982	3123.90
SARIMA(1,0,1)×(1,1,1) ₁₂	3558.85	18.2852	3118.40
SARIMA(2,0,1)×(1,1,1) ₁₂	3560.18	20.2977	3126.79
SARIMA(3,0,1)×(1,1,1) ₁₂	3559.91	22.3065	3123.78
SARIMA(1,0,3)×(1,1,1) ₁₂	3559.78	22.3061	3123.79
SARIMA(1,0,0)×(2,1,1) ₁₂	3555.24	18.2768	3238.72
SARIMA(0,0,1)×(1,1,1) ₁₂	3557.34	16.2722	3117.09
SARIMA(0,0,2)×(1,1,1) ₁₂	3559.31	18.2862	3119.80
SARIMA(0,0,3)×(1,1,1) ₁₂	3560.48	20.2984	3121.29
ASARIMA(0,1,1) ₁₂	3553.90	12.2455*	3116.94
ASARIMA(0,1,2) ₁₂	3551.61	14.2496	3186.91
ASARIMA(0,1,3) ₁₂	3554.10	16.2647	3218.03
ASARIMA(1,1,1) ₁₂	3555.91	14.2595	3116.73
ASARIMA(2,1,1) ₁₂	3550.12*	16.2555	3229.91
ASARIMA(1,1,2) ₁₂	3551.88	16.2596	3119.28
ASARIMA(2,1,2) ₁₂	3551.24	18.2676	3157.69

Using the Modified Box-Pierce statistic, all 18 of the models in Table II exhibited no serial correlation in the model residuals up to the 12th lag. When comparing models using AIC, it is shown that ASARIMA(2,1,1)₁₂ has the minimum AIC value, making it the preferable sub-class among SARIMA(p,d,q)I(P,D,Q)₁₂ and ASARIMA (P,D,Q)₁₂ models. The BIC also favoured ASARIMA over SARIMA, even

though AIC is the selected information criteria. Nevertheless, SSDFC favours SARIMA(1,0,0)×(1,1,1)₁₂, followed by ASARIMA(1,1,1)₁₂, when considering output performance metrics such prediction for 150 lead time.

TABLE III. FINAL ESTIMATES OF ASARIMA(2,1,1)₁₂ PARAMETERS

Type	Coef	SECoef	T	P
SAR 12	0.0047	0.0518	0.09	0.928
SAR 24	24	0.0519	-1.89	0.060
SMA 12	12	0.0234	40.66	0.000
Constant	1.0529	0.2195	4.80	0.000

Dissimilarities: 0 perennial, 1 annual of 12th order Original series 432; after differencing, 420 observations The remaining values are: SS

= 157,186 (3779) DF = 416 (not including backforecasts).

You can see that SAR at lag 24 and SMA lag 12 are both significant factors in the model, with a 5% and 10% significance level,

respectively, in Table III. A model known as ASARIMA(2,1,1)12 might be expressed as

$$\nabla_{12}X_t = 1.0529 + 0.0047\nabla_{12}X_{t-12} - 0.0981\nabla_{12}X_{t-24} + 0.9513u_{t-12} \quad (9)$$

TABLE IV. MODIFIED BOX-PIERCE (LJUNG-BOX) CHI-SQUARE STATISTIC

Lag	12	24	36	48
Chi-Square	13.4	24.2	45.5	53.4
DF	8	20	32	44
P-Value	0.099	0.058	0.058	0.0156

The result of Table IV shows that the probability of Modified Box-Pierce (Ljung-Box) Chi- Square statistic are all greater than 5% significant level, this indicates that

the residuals of the ASARIMA(2,1,1)12 are not correlated up to 48th lag. Hence the model is adequate.

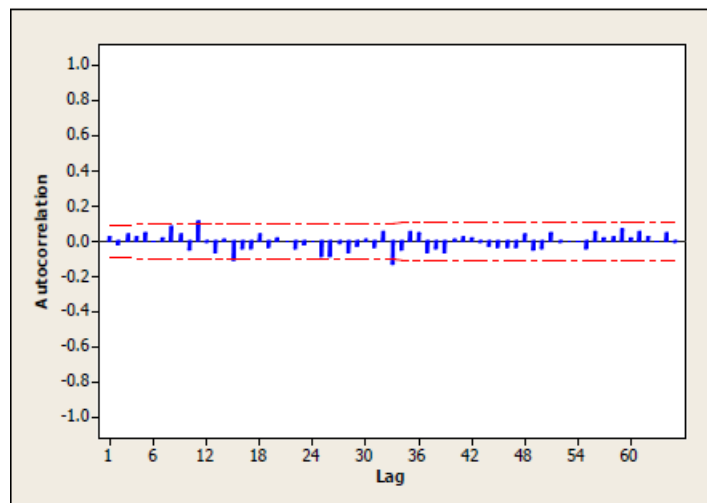


Fig.5. Plot of ACF of Residuals (with 5% significance limits for autocorrelations)

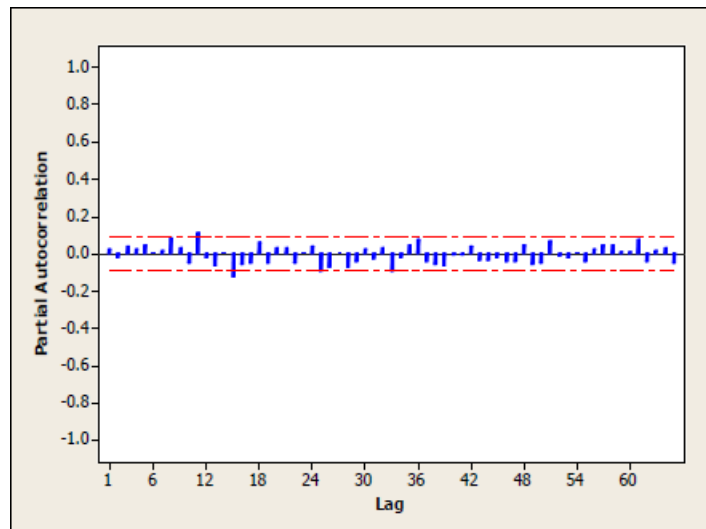


Fig.6. Plot of PACF of Residuals (with 5% significance limits for autocorrelations)

The ACF and PACF of residuals in Figure 5 and Figure 6 respectively for the Enugu rainfall data showed no significant spikes (the spikes are within the confidence limits) indicating that the residuals are uncorrelated. Therefore, the ASARIMA(2,1,1)12 model appears to fit well and can be used to make forecasts for Enugu monthly rainfall.

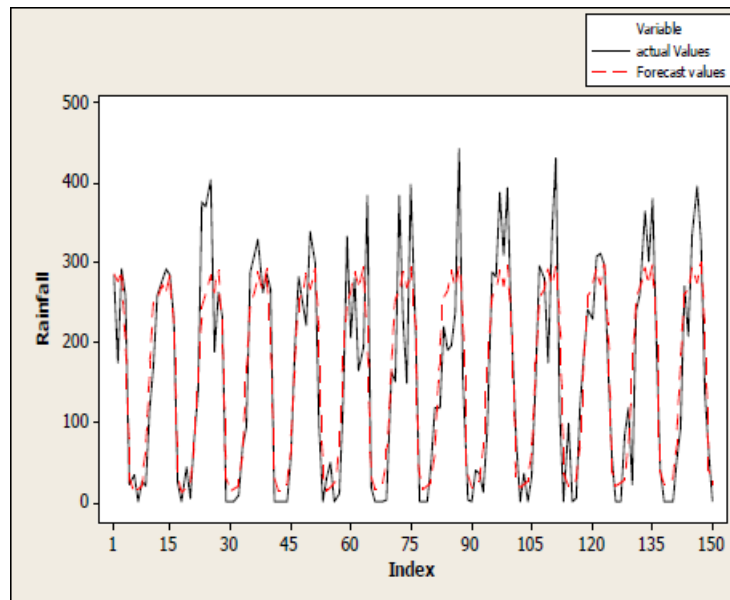


Fig.7. Time plot of forecast and actual values

The predicted values in Figure 7 were quite similar to the actual values. As a result, the fitted model seems to have done okay. Section B: Analysing the Findings The comparative analysis utilising AIC demonstrates that in a normal stationary time

series variable with seasonal nonstationary behaviour, as the EMR pattern of monthly rainfall in Enugu,

Better results were achieved by ASARIMA(2,1,1)12 compared to all the sub-

classes of the SARIMA(p,d,q)×(P,D,Q) model. With respect to the ASARIMA(2,1,1)₁₂ residuals, the Modified Box-Pierce (Ljung-Box) Chi-Square statistic shows no correlation up to the 48th lag. The prediction values are rather close, and the ACF and PACF of the model residuals are also uncorrelated, hence the fitted model is adequate.

This study adds a new dimension to the modelling of seasonal behaviour of variables that are considered regularly stationary (where $d = 0$) and seasonally nonstationary (where $D = 1$), in contrast to previous research that focused on the application of the SARIMA model introduced by Box and Jenkins (1979).

V. CONCLUSION

VI. Results show that ASARIMA(2,1,1)₁₂ subclass outperforms all SARIMA subclasses according to AIC in a normal stationary time series with seasonal nonstationary behaviour, such monthly rainfall data from Enugu. The study examined SARIMA and Adjusted SARIMA models in this context. Because of its capacity to decrease model parameter redundancy and sum of square errors, ASARIMA is therefore suggested for such time series patterns.

References

- [1] Afrifa-Yamoah E , I. I. Bashiru Saeed , A. Karim (2016) Sarima Modelling and Forecasting of Monthly Rainfall in the Brong Ahafo Region of Ghana, World Environment, Vol. 6 No. 1, 2, pp. 1-9.
- [2] Akpanta A. C, I. E. Okorie¹ and N. N. Okoye (2015). SARIMA Modelling of the Frequency of Monthly Rainfall in Umuahia, Abia State of Nigeria. American Journal of Mathematics and Statistics; 5(2): 82-88.
- [3] Alawaye A.I and A.N.Alao(2017). Time Series Analysis on Rainfall in Oshogbo Osun State, Nigeria, International Journal of Engineering and Applied Sciences (IJEAS); 4(7): Pp.35-37.
- [4] Amaefula, C. G..(2011). Optimal identification of subclass of autoregressive integrated moving average model using sum of square deviation forecasts criterion. International Journal of Statistics and System.2011; 6(1): Pp.35-40.
- [5] Amaefula, C. G.,(2018). Modelling Mean Annual Rainfall Pattern in Port Harcourt, Nigeria.EPRA International Journal of Research and Development (IJRD); 3(8): Pp.25-32.
- [6] Amaefula, C. G.,(2019). Modelling Monthly Rainfall in Owerri, Imo State Nigeria using SARIMA. EPRA International Journal of Multidisciplinary Research (IJMR); 5(11): Pp.197 – 206.
- [7] Box G.E.P and G.M.Jenkins(1976). Time Series Analysis, Forecasting and Control, Holden-Day: San Francisco.
- [8] Box, G. E. P. and G. M.Jenkins, (1994). “Time Series Analysis, Forecasting and Control”, 3rd Edition. Prentice Hall, ,1994

- [9] Dickey, D.A. and W.A., Fuller (1979). "Distribution of the Estimators for Autoregressive Time Series with a unit root," Journal of the American Statistical Association; 74: Pp.427-431.
- [10] Edwin A. and O. Martins (2014) Stochastic Characteristics and Modelling of Monthly Rainfall Time Series of Ilorin, Nigeria: Open Journal of Modern Hydrology; Vol. 4: Pp.67-69.
- [11] Elliott, G., T.J. Rothenberg and J.H. Stock (1996). Efficient test for an autoregressive unit root. *Econometrica*; 64(4): Pp.813-336.
- [12] Etuk H. E, U. I. Moffat and E. B. Chims (2013). Modelling Monthly Rainfall Data of Port Harcourt, Nigeria by Seasonal Box-Jenkins Methods, *International Journal of Sciences.*; 2(7): Pp 60-67.
- [13] Igwenagu, C. M., (2015). Trend Analysis of Rainfall Pattern in Enugu State Nigeria. *European Journal of Statistics and Probability*; 3(3): Pp.12-18.
- [14] Nirmala, M and S.M. Sundaram, (2010). A Seasonal ARIMA Model for Forecasting monthly rainfall in Tamilnadu. *National Journal on Advances in Building Sciences and Mechanics.* 1(2): Pp. 43-47.
- [15] Osarumwense, O.I., (2013). Applicability of Box Jenkins SARIMA Model in Rainfall Forecasting: A Case Study of Port-Harcourt South- South Nigeria. *Canadian Journal on Computing in Mathematics, Natural Sciences, Engineering and Medicine.* 4(1) :Pp.1-5.
- [16] Yusuf, F and I. L. Kane (2012). Modeling Monthly Rainfall Time Series using ETS state space and SARIMA models. *International Journal of Current Research.* 2012; 4(9): Pp. 195 -200.